Algebra II

11-5

Sums of Arithmetic and Geometric Series

Sum of a Finite Arithmetic Series :

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Find the sum of each arithmetic series.

$$\underbrace{1 + \underbrace{2 + 3 + 4 + 5 + \ldots + 99} + 100}_{101}$$

a_1 $n=1$ a_{100}

$$S_{100} = \frac{100(1+100)}{2} = \frac{100(101)}{2} = 5050$$

Sum of a Finite Geometric Series :

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Find the sum of each geometric series.

$\cdot 2 \quad \cdot 2$

$2 + 4 + 8 + 16 + \ldots + 1024$

$a_1 \quad a_2 \qquad\qquad a_n$

geometric: $r = 2$

$a_n = a_1 r^{n-1}$

$1024 = 2(2)^{n-1}$

$512 = 2^{n-1}$

$2^9 = 2^{n-1}$

$n = 10$

$S_{10} = \frac{2(1-2^{10})}{1-2}$

$= \frac{2(1-1024)}{-1}$

$= \frac{2(-1023)}{-1}$

$= \boxed{2046}$

Find the sum of each arithmetic series.

1) $n = 20$; $a_1 = 5$; $a_{20} = 62$

$S_{20} = \frac{n(a_1 + a_n)}{2}$

$= \frac{20(5+62)}{2}$

$= 10(67) = \boxed{670}$

Find the sum of each arithmetic series.

7) $\sum_{j=1}^{50} 3j + 2 = 5 + 8 + 11 + \cdots + 152$

$$S_{50} = \frac{50(5+152)}{2}$$

$$= 25(157)$$

$$= \boxed{3925}$$

Find the sum of each geometric series.

17) $\sum_{k=1}^{12} 2^{-k} = 2^{-1} + 2^{-2} + \cdots + 2^{-12}$

$$= \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{12}}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\frac{1}{2}} = \frac{\cancel{\frac{1}{2}}\left(1-\frac{1}{4096}\right)}{\cancel{\frac{1}{2}}}$$

$$= \boxed{\frac{4095}{4096}}$$

Find the sum of the following.

21) The first 20 positive integers ending in 3

$3+13+23+\cdots$

$a_{20}=a_1+d(n-1)$

$a_{20}=3+10(20-1)$

$=3+10(19)$

$=193$

$$S_{20}=\frac{20(3+193)}{2}$$

$=10(196)$

$\boxed{=1960}$

Pg 527
2-28 even

Pg 533
2-12 even